Power Budgeted Packet Scheduling for Wireless Multimedia

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Abstract—In this paper we profile a particular tradeoff between power budget and video quality that emerges in the transmission of multimedia packets over a wireless channel. These packets are due to arrive to a receiver at a particular time, so we consider a finite horizon problem over which multimedia data are transmitted. Due to the lossy nature of the wireless channel, however, not every packet can be successfully sent across the channel. Hence, each packet that is lost leads to distortion in the video that is experienced by the receiver. We suppose that there are \( M \) packets that must arrive at the receiver within \( N \) time steps, but that power limitations constrain the number of transmissions. At each time step, we may make a measurement of the wireless channel and decide whether or not to transmit a packet over the channel at that time. First we will suppose the times at which the channel state is sampled are spaced far enough apart so that the samples are i.i.d. Then we will continue by supposing that the channel state follows a Markov chain.

Index Terms—Dynamic Programming, Multimedia, Wireless, Power Control

I. INTRODUCTION

As the state-of-the-art advances in multimedia compression and streaming technologies, a number of technical challenges have also arisen related to transmitter power control and signal distortion. In particular, there is a tradeoff between power usage on the transmitter side versus quality of playout on the receiver side. That is, to mitigate adverse effects such as underflow of video content (resulting in disruption of the user experience) and distortion of frames, the transmitter must increase the power used to transmit data to the receiver. In this paper, we focus on one aspect of this tradeoff between power consumption and playout quality and determine an optimal policy for balancing this tradeoff with a dynamic programming approach. While the complete picture of communicating multimedia over lossy channels is more involved than the model presented here, we spotlight one particular type of power budget constraint and choose to address refinements to the model in future work.

Indeed, the space of wireless multimedia is a large one with many parameters and constraints. The specific area of power control is an area which has seen significant growth in recent years. Dynamic programming as well as heuristic methods are employed to achieve optimal or near-optimal performance for a given set of parameters. Especially noteworthy is [1], where a dynamic programming framework for performing rate-distortion optimized streaming was proposed. Key contributions offer solutions to manage the tradeoff between the delay of individual packets and the power spent in their transmission [2]. Other approaches, such as adaptive playout, have been used in related problems such as streaming audio and video over the Internet [3].

Optimization of media over a network has also seen attention, as in [4] where cross-layer optimization over wireless is studied, as well as [5] where dynamic programming is used to find quantization levels and perform rate control. For example, [6] develops a DP framework to decide which media units to transmit and which to discard in order to get rate-distortion optimized streaming. Some contributions utilize channel fluctuations to more effectively transmit video over wireless while still others use dependencies between media units. In [7], the authors jointly leverage power and playout control to achieve a certain playout quality yet minimally stress the wireless channel and battery.

In this work, we take a different approach to power control. We consider a finite horizon problem so that all packets that we send are scheduled to arrive before a certain time, a situation frequently encountered in applications such as video conferencing where large delays are not tolerated and all packets of each frame need to arrive within a time interval allowed for this frame. Over the duration of the problem, the number of times that packets may be sent is constrained. This corresponds to a commitment to use a certain amount of battery power over that time, but no more. Hence, if there are many opportunities remaining to make transmissions, the optimal policy will be more liberal and will attempt to send information even in suboptimal channel conditions. On the other hand, if the remaining chances to transmit are scarce, the policy will be very conservative and only send data when channel conditions are extremely good.

This approach to resource limited control was introduced in [8] where the two agents are to communicate the state of a random process from one to the other using a channel that may only be accessed a certain number of times. Work in this direction has also been done in the context of power limited surveillance/monitoring, where observations are expensive but
one would still like to track the state of a random process carefully [9].

Our paper is organized as follows: In Section 2 we describe our problem and model it mathematically. Section 3 provides a method for optimally making transmissions in the case that channel quality states are i.i.d. and Section 4 considers the case in which they evolve according to a Markov model. Section 5 provides numerical results and offers a discussion of them and finally Section 6 summarizes our findings and offers directions for future work.

II. MODEL

Let us begin by describing our problem in greater detail. We are interested in a scenario in which multimedia packets must be transmitted across a lossy wireless channel in discrete time. The packets are to be processed by the receiver and hence are due at a certain time. Therefore, we utilize a framework of finite-horizon control. This situation is seen in mobile video communication where the nature of the application is intolerable to packets arriving after some particular deadline. In this framework, we consider that a number of packets, \( M \), must be transmitted over the channel over \( N \) time steps. In these \( N \) time steps, the channel’s quality varies so that the probability of a successful transmission is changing over time.

Without limitations on the usage of this channel, the optimal strategy is straightforward: one would simply utilize every time step to send a packet. If acknowledgements are present to alert the transmitter of a failed transmission, one may retransmit as many of those as possible, since there are only \( M \) packets to transmit and \( N \) opportunities to do so. If acknowledgements are not present, one can retransmit those packets that were sent during the poorest channel conditions to maximize the expected number of packets sent.

The reality, however, is that there are limitations on the amount of power that may be utilized, especially in the context of a mobile environment. In light of this, we model an additional constraint on the number of times the channel may be utilized. Without loss of generality, we may take this limitation to be \( M < N \). If there are more opportunities to transmit than there are packets to send, packets can be retransmitted in the manner described above.

The channel in Fig. 1 represents one in which there is a limit on the number of times it may be used and an associated channel state at each time \( k \).

![Wireless Channel](image)

Fig. 1. Transmission of video packets over a wireless channel

We note that although we do not account for dependencies between video packets that are strung together, the cost structure can be modified and the state space may be augmented to account for this in the construction of an optimal policy.

A. Mathematical Formulation

This situation can be captured by considering a collection of possible channel states, \( S \), and each \( x \in S \) has an associated probability and channel quality state (as measured by the probability of a packet going through). We begin by considering the i.i.d. random process \( \{x_k\}^N_{k=0} \) to model the channel quality state of the wireless channel. Each \( x_k \) takes a value \( x \in S \) with probabilities \( p(x) \). The channel quality state shall be denoted by considering that each \( x \in S \) has a probability of success \( c(x) \in [0, 1] \).

Consider the class of transmission policies consisting of a sequence of functions

\[
\Pi = \{\mu_0, \mu_1, \ldots, \mu_{N-1}\}
\]

where each function \( \mu_k \) maps the information available to the controller at time \( k \) to an action \( u \) from the set \( A = \{\text{transmit, don't transmit}\} \) with the additional constraint that one may transmit at most \( M \) times.

We want to find a policy \( \pi^* \in \Pi \) to minimize the cost

\[
J^\pi_{(M,N)} = E \left\{ \sum_{k=0}^{N-1} c(x_k)I_{\text{packet-not-sent-at-time-k}} \right\}
\]

where the indicator function \( I \) is one if it is decided not to send the packet and zero otherwise, and where \( J^\pi_{(M,N)} \) is the cost-to-go when there are \( N \) time steps remaining and \( M \) opportunities to transmit information. The search for an optimal policy can also be written:

\[
J^\pi_{(M,N)} = \min_{\pi \in \Pi} J^\pi_{(M,N)}
\]

That is, we are minimizing the channel quality state of the channel for the time slots in which transmissions are not made, which is equivalent to maximizing the channel quality state of the channel over time during which transmissions are made.

The problem is trivial when \( M = N \) because in this case one can simply send a packet at every time step which results in zero cost (or full channel utilization). So we shall only consider the case in which \( M < N \).

After first studying the case in which the channel state \( \{x_k\}^N_{k=0} \) is described by a i.i.d. random process, we move on to a scenario in which it evolves according to the dynamics of a Markov chain.

III. I.I.D. CASE

We begin by modeling the channel states as coming from an i.i.d. distribution. Due to the fact that the past is decoupled from the future accumulated cost in the i.i.d. case, the optimal policy for time \( k \) needs only consider \( \{x_k, s_k, t_k\} \) where \( s_k \) and \( t_k \) are the number of packet transmissions left and the number of time slots remaining, respectively. For simplicity we will drop the subscript for \( s_k \) and \( t_k \).

We reformulate the problem so that we seek a policy such that we should transmit a packet if \( x_k \in \Lambda_{(s,t)} \) and not send a packet otherwise. \( \Lambda_{(s,t)} \) is some subset of the set of all possible channel states \( S \). We take a dynamic programming approach to this end.
From the DP equation we have,
\[
J^*(s,t) = \min_{\Lambda(s,t)} \left\{ P[x_k \in \Lambda(s,t)]J^*(s-1,t-1) + \sum_{x \in \Lambda(s,t)} p(x)c(x) \right\}
\]
This can be seen by noting that the first term of the minimization is the cost-to-go from the next stage if the channel is used in the current time step, the second term is this cost-to-go when the channel is not used and the third term is current stage cost. It can equivalently be written:
\[
J^*(s,t) = \min_{\Lambda(s,t)} \left\{ (1 - P[x_k \in \Lambda(s,t)])J^*(s-1,t-1) + \sum_{x \in \Lambda(s,t)} p(x)c(x) \right\}
\]
by considering the complement of the set \(\Lambda(s,t)\). Now we can plug in \(P[x_k \in \Lambda_c(s,t)] = \sum_{x \in \Lambda_c(s,t)} p(x)\) and rearrange terms to get:
\[
J^*(s,t) = \min_{\Lambda(s,t)} \left\{ - \left( J^*(s-1,t-1) - J^*(s,t-1) \right) \sum_{x \in \Lambda(s,t)} p(x) \right. \\
+ \left. \sum_{x \in \Lambda(s,t)} p(x)c(x) \right\} + J^*(s-1,t-1)
\]
which we can modify by combining like terms to get:
\[
J^*(s,t) = J^*(s-1,t-1) \\
+ \min_{\Lambda(s,t)} \left\{ \sum_{x \in \Lambda(s,t)} p(x) \left( c(x) - \left( J^*(s-1,t-1) - J^*(s,t-1) \right) \right) \right\}
\]
The minimization is over all possible subsets \(\Lambda(s,t)\), which is in general combinatorial and intractable. In this case, however, we can exploit the structure of the objective to conclude that for an optimal policy we must have:
\[
x \in \Lambda_c(s,t) \Leftrightarrow c(x) < J^*(s-1,t-1) - J^*(s,t-1)
\]
It is clear that \(0 \leq J^*(s-1,t-1) - J^*(s,t-1) \leq \max_{x \in S} c(x)\). Replacing our result into the previous equations, we finally get
\[
J^*(s,t) = J^*(s-1,t-1) \\
+ \sum_{x \in S} p(x) \left( c(x) - \left( J^*(s-1,t-1) - J^*(s,t-1) \right) \right)
\]
This equation holds for \(0 < s < t \leq N\). We also must consider the following boundary conditions:
\[
J^*_0 = t \left( \sum_{x \in S} p(x)c(x) \right), \quad J^*_N = 0
\]
which follow from the facts that when \(s = 0\), we accrue a cost at every stage equal to the average channel quality state.

A. Structure of the solution

Using the results above, a table (Fig. 2) of optimal average errors may be constructed offline and referenced as decisions are being made in real time. That is, for each \((s, t)\), we may determine a corresponding \(\Lambda^*_c\) using (4).

![Fig. 2. Offline dynamic programming solution structure](image)

Once these values are obtained, one may apply the policy
\[
u(s,t) = \begin{cases} 
0 & \text{if } x \in \Lambda^*_c(s,t) \\
1 & \text{otherwise}
\end{cases}
\]
where an action of \(u = 0\) corresponds to no transmission and an action of \(u = 1\) corresponds to transmission of the packet.

We finally note that although these computations involve taking sums over a possibly large state space, the sums correspond to inner products which may be doing efficiently by using a “kernal trick”.

IV. Markov Case

We turn to the case in which the quality of the channel at each time, \(x_k\), is not described by an i.i.d. random process, but rather as a discrete-time Markov chain. We shall now consider a scenario in which the channel state is still described by a finite collection of states, but the transitions between them follow Markov chain dynamics. In addition to the notation developed above, we also introduce a transition matrix \(P\) which captures the probability of transitioning from one state to another. We consider the state space \(S\) to be ordered so that we can map the rows and columns of \(P\) to specific states. For convenience we also introduce the vector \(c\), which is a vector of the probabilities of successful transmission, \(c(x)\), for each channel state, \(x\).

In the following development, we again seek a policy \(\pi^* \in \Pi\) to minimize the cost
\[
J^*_M = E \left\{ \sum_{k=0}^{N-1} c(x_k) \text{packet not sent at time } k \right\}
\]
We proceed to construct the solution using backwards induction. We begin with $t = 1$, which corresponds to one unit of time remaining in the problem, and then continue for $t = 2, 3, \ldots$ until we are able to determine a recursion. As we build backwards in time (and forward in $t$), we let $s$ vary and keep track of the cost $J_{s,t}(x)$ where $x \in S$ is the currently observed channel state. (Note the abbreviation from $x_{(N-t)}$.

For $t = 1$, we can either have $s = 0$ or $s = 1$. These costs, respectively, are (in vector form)

$$J_{(0,1)} = c \quad J_{(1,1)} = 0$$

since not having a transmission opportunity means that we accu-

rate a cost equal to the probability of a successful transmission

from that state.

Moving on to $t = 2$, the values of $s$ can range from $s = 0$,
$s = 1$ or $s = 2$. For $s = 0$ we have

$$J_{(0,2)} = c + Pc$$

since we accrue a cost equal for the current stage as well as

the next stage, when the state has changed according to the

transition matrix $P$. When $s = 1$, there are two choices: use

an opportunity to make an observation so that $u = 1$ or do not

observe, in which case $u = 0$. These choices can be denoted

with superscripts above the cost function for each stage:

$$J_{(0,2)}^{(1)} = c \quad J_{(1,2)}^{(1)} = Pc$$

For $u = 0$, we accrue error for the current time slot and no

error afterwards. When an observation is made, no error is

accrued for the current time slot $N-2$, but there is error in the

next time slot which depends on the current observation.

We now introduce some new notation:

$$\Delta_{(1,2)} = J_{(1,2)}^{(0)} - J_{(1,2)}^{(1)} = (I - P)c$$

so that if $\Delta_{(1,2)}(x) \leq 0$, then we should not make a

transmission, whereas we should make one if $\Delta_{(1,2)}(x) > 0$.

We proceed now by defining sets $\tau_{(1,2)}$ and $\tau_{(1,2)}^{*}$ such that

$$x \in \tau_{(1,2)} \Rightarrow \Delta_{(1,2)}(x) \leq 0$$

$$x \in \tau_{(1,2)}^{*} \Rightarrow \Delta_{(1,2)}(x) > 0$$

and we also define an associated vector $1_{(1,2)} \in \{0,1\}^S$

$$1_{(1,2)}(x) = \begin{cases} 1 & \text{if } x \in \tau_{(1,2)}^{*} \\ 0 & \text{otherwise} \end{cases}$$

Moving on to $s = 2$, we have $J_{(2,2)} = 0$, since there are

as many opportunities to make transmissions as there are

remaining time slots. We continue with $t = 3:

$$J_{(0,3)} = c + Pc + P^2c$$

since there are three time slots for which cost is accrued. For $s = 1$, we again have a choice of $u = 0$ and $u = 1$. For $u = 0$, we accrue a cost for the current stage, and then count

the future cost depending on the current state:

$$J_{(1,3)}^{(0)}(x) = c(x) + \sum_{y \in S} P(x,y) \left( J_{(1,2)}^{(0)}(y) + (1 - 1_{(1,2)}(y)) J_{(1,2)}^{(1)}(y) \right)$$

and combining terms gives us

$$J_{(1,3)}^{(0)}(x) = c(x)$$

whereafter substituting the value of $J_{(1,2)}^{(1)}(x)$ and putting

things in vector form gives us:

$$J_{(1,3)}^{(0)} = c + P^2c + Pdiag(1_{(1,2)})\Delta_{(1,2)}$$

Now we consider the $u = 1$ case:

$$J_{(1,3)}^{(1)} = Pc + P^2c$$

since there no current cost, but the last two stages produce

costs that depend on the currently observed channel state. We

now write the expression for $\Delta_{(1,3)} = J_{(1,3)}^{(0)} - J_{(1,3)}^{(1)}$:

$$\Delta_{(1,3)} = c - Pc + Pdiag(1_{(1,2)})\Delta_{(1,2)}$$

Continuing with $s = 2$,

$$J_{(2,3)}^{(0)} = c + 0$$

whereas for $u = 1$,

$$J_{(2,3)}^{(1)} = 0 + \sum_{y \in S} P(x,y) \left( 1_{(1,2)}(y) J_{(1,2)}^{(0)}(y) + (1 - 1_{(1,2)}(y)) J_{(1,2)}^{(1)}(y) \right)$$

where we have accounted for the cost stage by stage: in the

current stage, no error is accrued since an observation is made

but future costs depend on the observation that is made. That

is, future costs depend on whether the next state $x_{(N-2)}$ is

observed to be in the set $\tau_{(1,2)}$. Averaging over these, we

obtain the expression above. Combining like terms, we arrive at:

$$J_{(2,3)}^{(1)} = P^2c + Pdiag(1_{(1,2)})\Delta_{(1,2)}$$

We use these expressions to get $\Delta_{(2,3)}$.

$$\Delta_{(2,3)} = c - P^2c - Pdiag(1_{(1,2)})\Delta_{(1,2)}$$

Finally, letting $s = 3$, we easily see: $J_{(3,3)}^{(0)} = 0$. This process

can be continued for $t = 4, 5, \ldots$ For each stage $(s, t)$, we

may determine $J_{(s,t)}^{(0)}$ and $J_{(s,t)}^{(1)}$. These costs then allow us to
determine when we should make an observation in the process and when we should not. The implementation of this policy is detailed in the following subsection.
A. Recursions

We now present a method for constructing an optimal policy. We do this by storing for each \((s, t)\) a subset of \(S\), denoted by \(\tau^c_{(s, t)}\), which is the set of last observed states for which we do not use an opportunity to view the process when we are at stage \((s, t)\). That is, if the currently seen channel state \(x\) is in the set \(\tau^c_{(s, t)}\), there are \(s\) opportunities remaining to make transmissions and there are \(t\) time slots remaining in the horizon then we should not make a transmission at this time. On the other hand, if \(x \not\in \tau^c_{(s, t)}\) then we should make a transmission at stage \((s, t)\) and accrue zero cost for that stage.

More precisely, an optimal policy \(\pi^*\) is given by

\[
\pi(x, s, t) = \begin{cases} 
0 & \text{if } x \in \tau^c_{(s, t)} \\
1 & \text{otherwise}
\end{cases}
\]

Let us introduce three vector valued functions: \(F_{(s, t)} \in \mathbb{R}^S\) and \(\mathbf{1}_{(s, t)} \in \{0, 1\}^S\). We fill in values for these functions by using the following recursions:

\[
F_{(s, t)} = P(F_{(s-1, t-1)} + \mathbf{1}_{(s-1, t-1)} \Delta_{(s-1, t-1)})
\]

\[
\Delta_{(s, t)} = c + P(F_{(s, t-1)} + \text{diag}(\mathbf{1}_{(s, t-1)}) \Delta_{(s, t-1)}) - F_{(s, t)}
\]

\[
\mathbf{1}_{(s, t)}(x) = \begin{cases} 
0 & \text{if } \Delta_{(s, t)}(x) > 0 \\
1 & \text{otherwise}
\end{cases}
\]

for \(1 < s < t < N\). We also have the boundary conditions

\[
F_{(1, t)} = 0, \quad F_{(1, 1)} = \sum_{m=1}^{t-1} P^m c, \quad \Delta_{(1, t)} = c
\]

These recursions allow us to determine the sets \(\tau^c_{(s, t)}\) for \(s, t\) in the bounds specified, which in turn defines our optimal policy. Specifically, we assign

\[
x \in \tau^c_{(s, t)} \iff \Delta_{(s, t)}(x) \leq 0
\]

We conclude by giving expressions for the cost-to-go from any particular state when a particular action \(u \in \{0, 1\}\) is taken. The superscripts denote whether or not an observation will be made in the current stage.

\[
J^{(0)}_{(s, t)} = c + P(F_{(s, t-1)} + \text{diag}(\mathbf{1}_{(s, t-1)}) \Delta_{(s, t-1)})
\]

\[
J^{(1)}_{(s, t)} = F_{(s, t)}
\]

Observe that \(\Delta_{(s, t)}\) is the difference between these two quantities. Hence, \(\Delta_{(s, t)}\) can be used to decide whether or not to make an observation in the current time step.

V. NUMERICAL RESULTS

Let us now consider a sample problem in which we must transmit video data over an i.i.d. wireless channel. We suppose that there are three \((1,2,3)\) channel quality states and that the packets are all due within 20 time steps. The following values are used:

\[
c(x) = (0.1 \ 0.5 \ 0.9)^T, \quad p(x) = (0.7 \ 0.2 \ 0.1)^T
\]

Note that for the most part, the probability of successful packet transmission is quite low (0.1), but on occasion the channel has decent or very good quality.

A. Throughput

We plot the expected number of successfully transmitted packets over the horizon \(N = 20\) when using the algorithm above as we vary the number of packets that may be transmitted. In the notation developed above, we plot \(c^T P - J^{(1)}_{(1, N)}\) as we vary \(s\). We also plot a “content aware” heuristic which sends the packets over the horizon in such a way that all the content has been attempted to be transmitted without concern for channel conditions, along with a "channel aware" heuristic which only transmits if the channel is not in the worst state, but does not necessarily send all packets.

![Plot of expected number of successful packets vs. number of transmissions](image)

Fig. 3. Plot of expected number of successful packets vs. number of transmissions

There is a region in which the channel aware approach performs almost as well as the optimal policy, specifically when the number of times the channel can be used is small compared with the horizon of the problem. After a point, however, its performance drops off because it does not utilize the channel when conditions are poor, but such a policy might be necessary sometimes. On the other hand, the content aware approach does well when there are almost as many transmissions as the horizon of the problem. If this is not the case, however, it does not differentiate between a good or poor channel and results in more lost packets.

B. Thresholds

Let us also consider a plot of the optimal thresholds for transmitting a packet. That is, we vary \(s\) and plot the lowest channel condition which will result in a transmission. In this simple example there are only three states to choose from \((0.9, 0.5 \text{ or } 0.1)\). It can be seen in Fig. 4 that the thresholds are decreasing since having more battery power results in more liberal usage of the channel. We also note that when \(s = t\) (we can transmit at every time step), the threshold for transmission drops to zero since it is always optimal to transmit.
advantage to using the optimal policy. (and more varied) the channel conditions, the greater the curve for a linear “content aware” approach. Hence, the worse the channel gets “good” channel, as well as for \((p = 0.7, 0.3), c = (0.1, 0.9)\) which represents a “bad” channel. The worse the channel gets, the higher the optimal curve lies above the corresponding curve, which represents a “bad” channel. The worse the channel gets, the higher the optimal curve lies above the corresponding curve, which represents a “bad” channel.

C. Comparisons

We finally note that the plot shapes depend greatly on the probabilities of channel state and the probabilities of successful transmission. In Fig. we plot the optimal tradeoff curve for \(p = (0.1, 0.9), c = (0.1, 0.9)\), which represents a “good” channel, as well as for \(p = (0.7, 0.3), c = (0.1, 0.9)\) which represents a “bad” channel. The worse the channel gets, the higher the optimal curve lies above the corresponding curve for a linear “content aware” approach. Hence, the worse (and more varied) the channel conditions, the greater the advantage to using the optimal policy.

VI. Conclusions

In this paper, we have examined a problem in communication of multimedia data over lossy channels when there are limits on channel usage. We mathematically model this as a finite horizon control problem and utilize a dynamic programming framework to develop policies that maximize throughput over the channel for i.i.d. channel states as well as Markov chains. We see that a thresholding policy on the channel state is optimal, and decisions depend on the number of opportunities remaining to transmit as well as the number of time slots left in the problem.

The resulting performance utilizes the channel when it is very good but also keeps track of the amount of time left in the process so that thresholds dynamically change according to the observed channel states and previous actions. This combination yields better performance than can be obtained by heuristics emphasizing a channel-aware approach or a content-aware approach, as can been seen in the previous section.

There are many related problems and extensions to consider in future work. One direction may be to investigate a scenario in which channel states still evolve according to a Markov model, but the state is unknown to the decision maker until a transmission is made.

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